national accelerator laboratory

EXP-39
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ACCELERATOR EXPERIMENT--4th Order Resonance in the Main Ring

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Experiment

An erratic beam loss is observed in the parabola region of the main ring acceleration cycle. In this region, the beam undergoes large variation of the closed orbit and the tune. The hypothesis that the beam loss might be caused by the crossing of some resonances has some proof in the following experiment.

The horizontal beam position is observed at the following pair of pick-up stations:

#1 at HF46 where $\beta_1 = 95.7\text{m}$ $\alpha_1 = 1.90$

#2 at HF48 where $\beta_2 = 97.0\text{m}$ $\alpha_2 = 0.46$

The transfer matrix from F46 to F48 is (in meters and radians)

$$\begin{pmatrix} 2.1369 & 91.057 \\ -0.013277 & -0.097797 \end{pmatrix}$$

which corresponds to a tune of ~20.27.

The actual tune, measured with small induced amplitude is 20.19.

Oscillograms of the output of both detectors have been taken overimposed in such a way that they show simultaneously the beam position at both locations every turn. At one edge of the oscillogram, it is visible, the closed orbit position just before the beam is pinged; then the pinger is applied, and finally, over the largest part of the picture, the induced coherent betatron oscillations are shown. The tune clearly corresponds to 20.25. At the same time, beam loss is observed as a consequence of the pinging action.

Call (x_1, p_1) and (x_2, p_2) the canonically conjugated variables of the coherent oscillations, respectively, at detectors #1 and #2. x_1 and x_2 are measured from the oscillograms. For this purpose, we first subtracted the closed orbit deviation and then transformed the reading from the oscillograms in length unit by means of the available calibration data. The other coordinates p_1 and p_2 are then calculated by means of the matrix (1).

Ignoring the first ~10 turns after the pinging, we have been tracking the motion of the beam in the phase plane of coordinates

$$\frac{x}{\sqrt{\beta}} \text{ and } \sqrt{\beta}p + \alpha\frac{x}{\sqrt{\beta}} \qquad (2)$$

turn after turn. The results of the tracking are shown in Figs. 1 and 2. The points are filling areas at the four corners of a diamond region. Because of the choice of the normalized coordinates (2), the diamond was expected to be exactly a square centered to phase plane. The fact that it is not exactly a square and is not centered is likely due to the reading errors from the oscillograms, to the fact we have used a matrix with a slightly different tune, and to the "not-so-accurate" knowledge of the function α and β.

Anyway, approximating the diamond with the closest square, we have for the half diagonal $\sqrt{I}$, and the area S of the square

$$I = 0.2 \times 10^{-3} \text{ cm}$$
$$S = 1.3\pi \text{ mm} \times \text{mrad}.$$

If we make the assumption that the four corners of the diamond in Fig. 1 and Fig. 2 are the four unstable fixed points of the fourth resonance, we can explain the beam loss since the apparent emittance of the pinged beam is likely larger than S.

<u>The Theory of the 4th Order Resonance</u>

The equations of motion in the angle-action variables (I,Ψ) are, when we take into account only octupole field ($\kappa = 3$):

$$\begin{aligned} I' &= -2\nu_o \sum_{n,m} I^2 b_{3m} f_{3n} e^{i(n\Psi+m\phi)} \\ \Psi' &= \nu_o - \nu_o \sum_{n,m} I b_{3m} g_{3n} e^{i(n\Psi+m\phi)} \end{aligned} \qquad (3)$$

where prime denotes derivation with respect to ϕ, and

$$f_{3n} = -i\frac{n}{4}g_{3n}$$

$$g_{3n} = \frac{1}{\pi}\int_0^{\pi} \cos^4 \Psi \cos n\Psi d\Psi$$

$$b_{3m} = \frac{1}{2\pi}\int_{-\pi}^{+\pi} \beta^3 a_3 e^{-im\phi} d\phi$$

$$a_3 = \frac{B'''}{6B\rho}$$

with $B\rho$ the magnetic rigidity.

Take in (3) only the slow varying turns

$$n = 4 \qquad m = -81$$
$$n = -4 \qquad m = 81$$
$$n = 0 \qquad m = 0$$

we obtain

$$I' = -4\nu_o\, I^2 g_{34} |b_{3,81}| \sin \left(4\Psi - 81\phi - \phi_{3,81}\right)$$

$$\Psi' = \nu_o - \nu_o I\, b_{30} - 2\nu_o\, I g_{34} |b_{3,81}| \cos \left(4\Psi - 81\phi - \phi_{3,81}\right)$$

where

$$g_{30} = 3/8 \qquad g_{34} = 1/16$$

and

$$b_{3,81} = |b_{3,81}|\, e^{i\phi_{3,81}}.$$

The transformation in the rotating coordinate

$$X = 4\Psi - 81\phi - \phi_{3,81}$$

gives

$$I' = -\frac{\nu_o}{4} I^2 |b_{3,81}| \sin X$$

$$X' = (4\nu_o - 81) - \frac{3}{2}\nu_o I\, b_{30} - \frac{\nu_o}{2} I |b_{3,81}| \cos X. \tag{5}$$

The coordinates of the fixed points are obtained by setting

$$I' = 0 \qquad \text{and} \qquad X' = 0.$$

From the first of Eq. (5) we have

$$X_1 = 0 \qquad\qquad X_2 = \pi$$

and from the second

$$I_1 = \frac{2}{\nu_o} \frac{4\nu_o - 81}{3b_{30} + |b_{3,81}|}$$

$$I_2 = \frac{2}{\nu_o} \frac{4\nu_o - 81}{3b_{30} - |b_{3,81}|}$$

By inspecting the motion in proximity of the fixed points and since I is by definition a positive quantity, we derive the following general results:

$$\text{(i)} \quad \boxed{3b_{30} > |b_{3,81}|}$$

There are no fixed points (the motion is generally stable) for $4\nu_o - 81 < 0$.

There are four stable and four unstable fixed points for $4\nu_o - 81 > 0$.

The coordinate of the unstable point is

$$I = \frac{2}{\nu_o} \frac{4\nu_o - 81}{3b_{30} + |b_{3,81}|}$$

$$\text{(ii)} \quad \boxed{3|b_{30}| < |b_{3,81}|}$$

There are only four unstable fixed points for whatever ν_o.

The coordinate of the unstable point is

$$I = \frac{2}{\nu_o} \frac{4\nu_o - 81}{3b_{30} + |b_{3,81}|} \qquad \text{for} \qquad 4\nu_o - 81 > 0$$

or

$$I = \frac{2}{\nu_o} \frac{4\nu_o - 81}{3b_{30} - |b_{3,81}|} \qquad \text{for} \qquad 4\nu_o - 81 < 0$$

$$\text{(iii)} \quad \boxed{3b_{30} < -|b_{3,81}|}$$

There are no fixed points (the motion is generally stable) for $4\nu_0 - 81 > 0$.

There are four stable and four unstable fixed points for $4\nu_0 - 81 < 0$.

The coordinate of the unstable point is

$$I = \frac{2}{\nu_0} \frac{4\nu_0 - 81}{3b_{30} - |b_{3,81}|} .$$

Combination of the Theory and the Experiment

From the experiment it is $4\nu_0 - 81 = -0.24$, so that we have to exclude case (i) above reported. From the other experiment data we derive, also, that

$$|b_{3,81}| - 3b_{30} = 120 \text{ cm}^{-1}.$$

And this is, probably, all the information we can get from the actual experiment.

At most, we can consider the following two extreme cases:

(a) $|b_{3,81}| >> 3|b_{30}|$, or

$$|b_{3,81}| \sim 120 \text{ cm}^{-1}.$$

If we calculate the driving term by using (4) and by spreading an rms octupole among the 240 quadrupoles in the ring, we obtain

$$\langle B''' \rangle_{rms} = 200 \text{ G/in}^3.$$

This is a very large number: about 40 times that we can derive from field measurement data (TM-403)

(b) $|b_{3,81}| << 3|b_{30}|$, or

$$b_{30} = -40 \text{ cm}^{-1}.$$

If, again, we use (4) and we spread an average octupole among the 240 quadrupoles we obtain

$$\langle B''' \rangle_{av} = 6\ \mathrm{G/in}^3 .$$

This number, also, is too large and not consistent with the field measurement data.

Suggestion

It is advisable to repeat the same experiment but by approaching the resonance from upper values of the tune.

This should give more information about b_{30} and $|b_{3,81}|$ independently.

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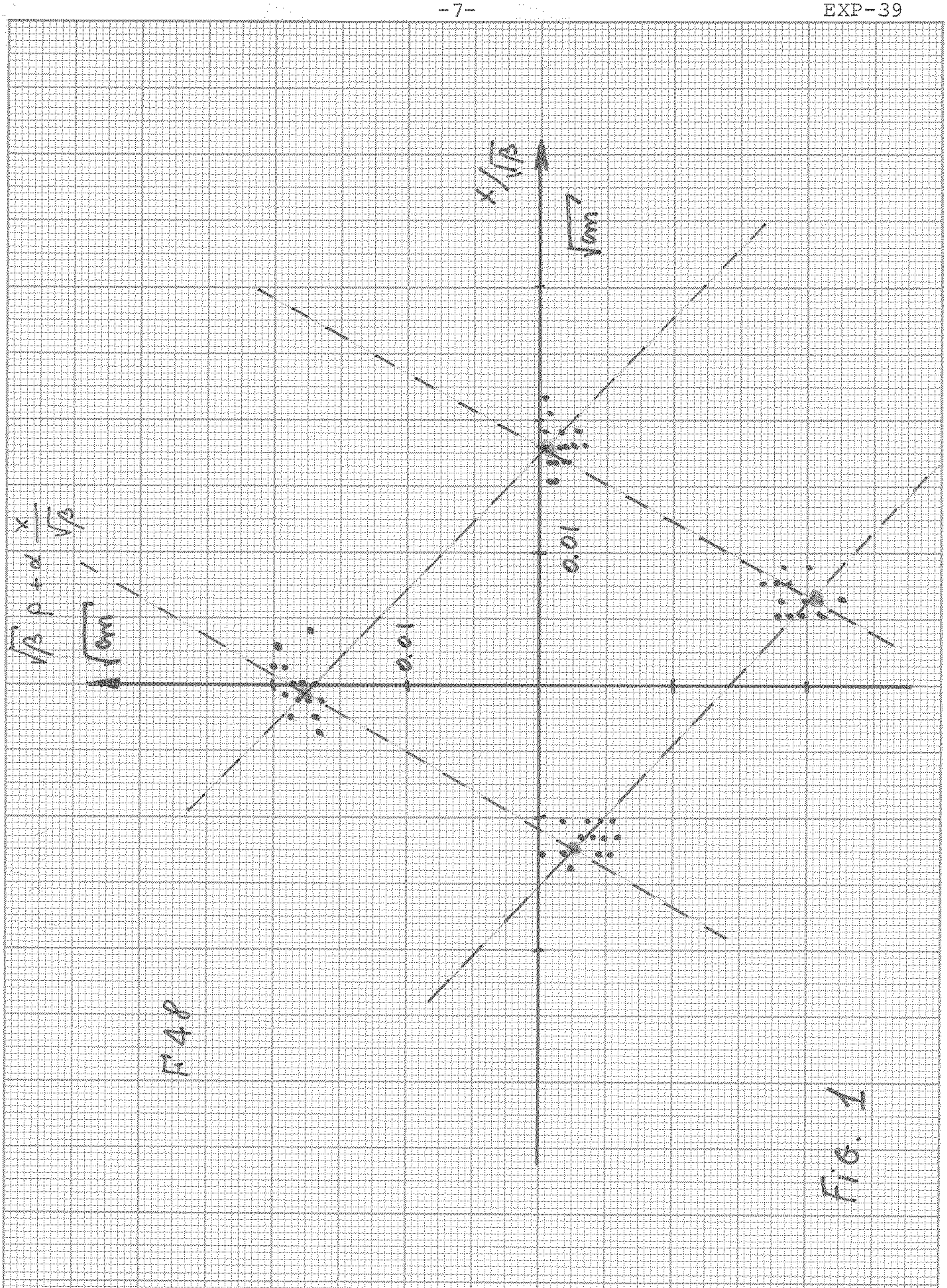
$x/\sqrt{\beta}$
$\sqrt{cm}$
0.01
0.01
$\sqrt{\beta}\,\rho + \alpha \frac{x}{\sqrt{\beta}}$
$\sqrt{cm}$
F 48

FIG. 1

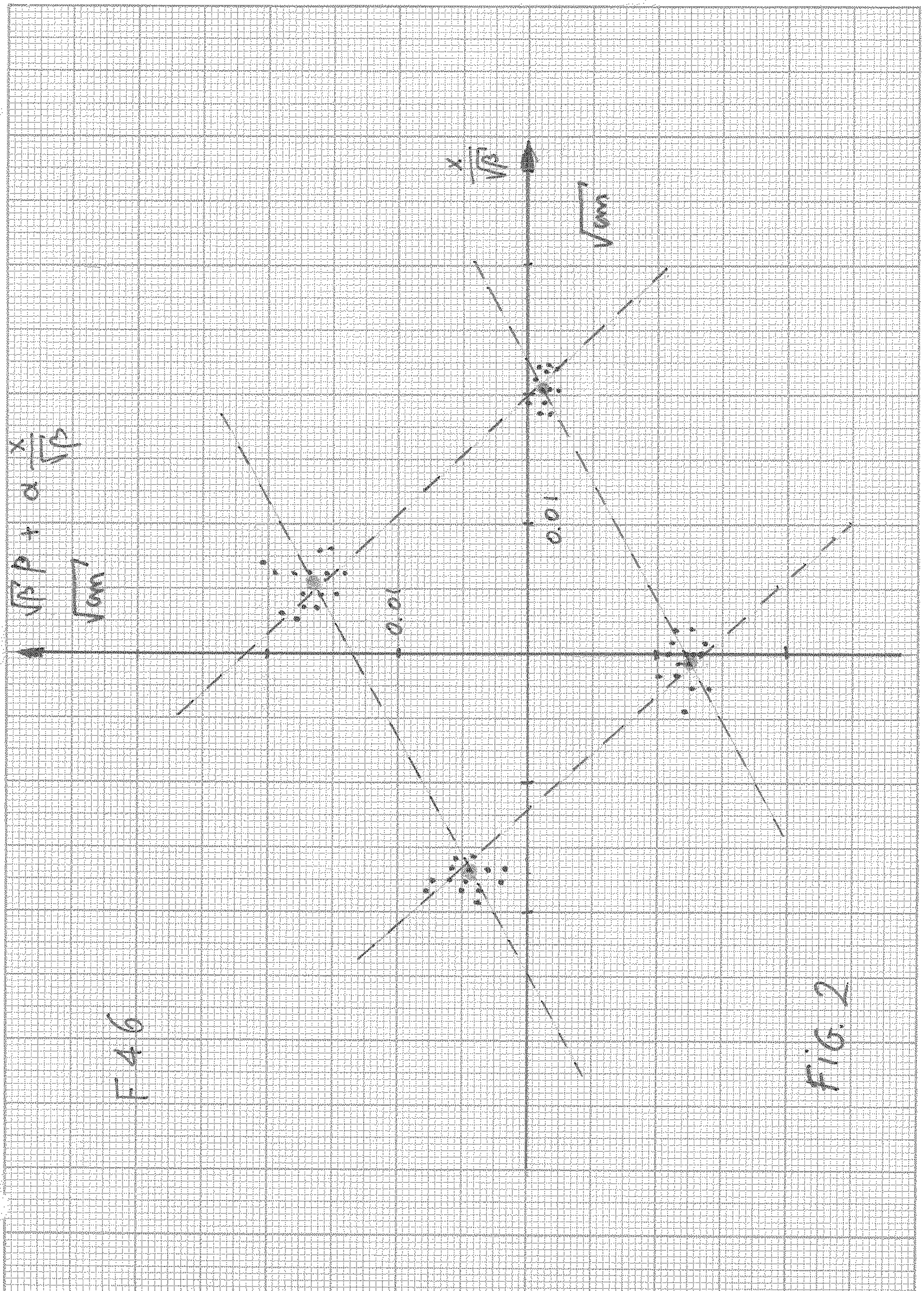
$\frac{x}{\sqrt{\beta}}$
$\sqrt{cm}$
0.01
0.01
$\sqrt{\beta}\,p + \alpha \frac{x}{\sqrt{\beta}}$
$\sqrt{cm}$
F 4 6

FIG. 2